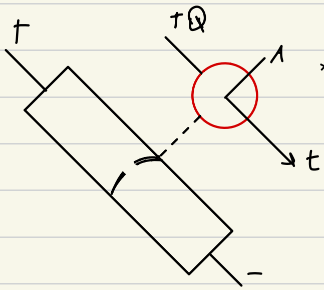
Current, Power, and Resistance

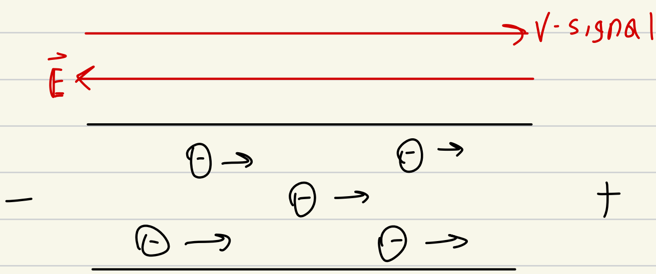
Electric Current (I) is the rate of the flow of charge (Q) through a cross section (A) in a unit of time (t)



*Zoom in of cross section

$I = Q/t$

• One Ampere (A) is charge flowing at the rate of one Coulomb per second (C/s)



- Electrons “drift” in wire - Slow Drift velocity (Ex. 0.000002 m/s)
- Thermal velocity - (Fast but “random”) (Ex. 1.57 million m/s)
  - Collision velocities
- The electrons bounce around by colliding with different molecules
- E-Field goes back, and since electron, E-Force is to the right

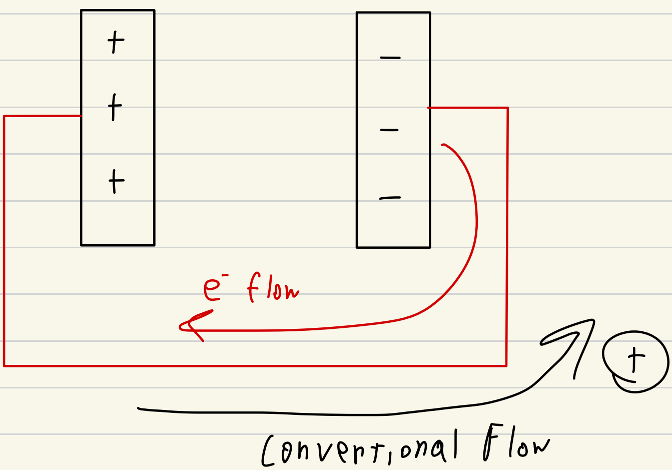
Ex. Electric current in a wire is 6A. How many electrons flow past a given point in a time of 3 seconds.

$I = 6A$ $\quad$ $6 = q/3$ $\quad$ $18/1.6 \times 10^{-19} = \boxed{1125 \times 10^{20} e^-}$

$I = q/t$ $\quad$ $q = 18 C$

Conventional Current:

- Imagine a capacitor (two conducting plates with opposite charge) that is allowed to discharge
- <u>Electron Flow</u> - The direction of electron flow from - to +
- <u>Conventional Current</u> - the motion of +q from - to +



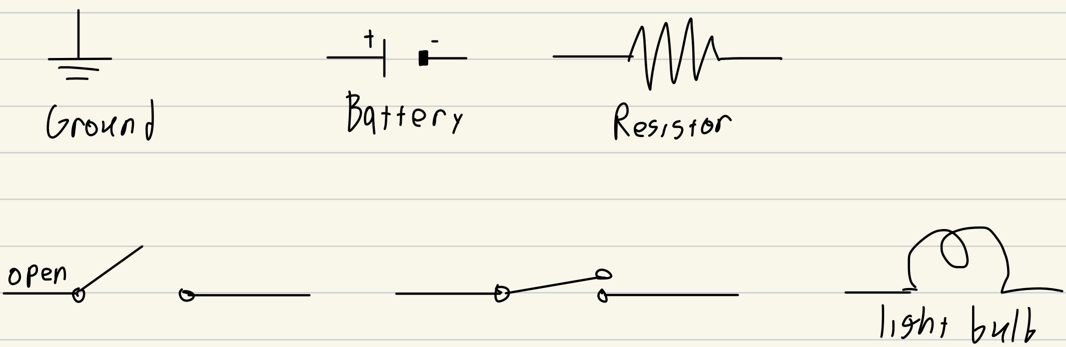
- Electric fields and potential are defined in terms of +q, so we will assume conventional current (even if electron flow is actual flow)

Electromotive Force:

- A source of Electromotive force (emf) is a device that used chemical, mechanical or other energy to provide the potential difference necessary for electric current.
    - Ex. Power lines, battery, wind generator

Electrical Circuit Symbols:

- Electrical Circuits: often contain one or more resistors grouped together and attach to and energy source, such as a battery



Electric Resistance:

Suppose we apply a constant potential different of 4V to the ends of geometrically similar rods of, say: steel, copper, and glass

- Materials have an electric resistance (R) property

Ohm's Law:

- Current I through a given conductor is directly proportional to the potential difference V between its end points

$$I \propto \Delta V$$

$$I = \frac{\Delta V}{R} \,;\quad \Delta V = IR \,;\quad R = \Delta V / I$$

Ex2. When a 3V battery is connected to a light, a current of 6mA is observed. What is the resistance of the light filament

$$R = \Delta V / I = 3 / .006 = 500\ \Omega$$

$$\Omega = \frac{1V}{1A}$$

Factors Affecting Resistance:

- The length L of the material. Longer materials have greater resistance

$1L$ — 1Ω $\qquad$ $2L$ — 2Ω

- Cross-sectional area A of the material. Larger areas offer LESS resistance
- The temperature T of the material. The higher temperatures usually result in higher resistance
- Kind of material. Iron has more electrical resistance than a geometrically similar copper conductor

Resistivity of a Material:

- The resistivity (*p*) is a property of a material the determines its electrical resistance R.

$$R = \rho \frac{L}{A} \quad \text{or} \quad \rho = \frac{RA}{L}$$

Ex. What length L of copper wire is required to produce a 4mOMEGA resistor? Assume the diameter of the wire is 1mm and that the resistivity *p* of copper is 1.72e-8 OMEGA x m.

$$A = \frac{\pi D^2}{4} = \frac{\pi(.001)^2}{4} = 7.85 \times 10^{-7}\ m/s^2$$

$$L = \frac{RA}{\rho} = \frac{(.004)(7.85 \times 10^{-7})}{1.72 \times 10^{-8}} = .163m$$

Temperature Coefficient:
For most materials, the resistance R changes in proportion to the initial resistance R_0 and to the change in temperature (DELTA)t.

$\Delta R = a R_0 \Delta t$ or $\rho(T) = \rho_0 + \rho_0 \alpha \Delta T$

- Ex.
  - The resistance of a copper wire is 4mOMEGA at 20 degrees Celsius. What will be its resistance if heated to 80 degrees Celsius. a= .004/ C degrees

$R_0 = 4 m\Omega$ $\Delta t = 80°C - 20°C = 60C°$
$\Delta R = a R_0 \Delta t$; $\Delta R = (.004/C°)(4m\Omega)(60C°)$

$\Delta R = 1.03 m\Omega$ $R = R_0 + \Delta R$
$R = 4m\Omega + 1.03m\Omega$

Electric Power:
- Electric power *P* is the rate at which electric energy is expended, or work per unit of time
  - Work = q(potential difference)

$P = \frac{W}{t} = \frac{q \Delta V}{t}$ & $I = q/t$

$\Delta V = IR$
$P = \Delta V (I)$
$P = I^2 R$
$P = \Delta V^2 / R$

A power is rated at a 9A when used with a circuit that provides 120-V. What power is used in operating this tool?

$P = \Delta V I = (120)(9) = 1080\,W$

A 500W heater draws a current of 10A. What is resistance?

$R = P/I^2 = \frac{500}{10^2} = 5\Omega$

How much energy does a 60W bulb consume in a week if it stays lighted for 12 hrs a day.

$P = 60W \quad t = 302400s$

$P = E/t \rightarrow E = P \times t = 18144000J$

We can express the power of the light bulb in KiloWatts instead of Watts

- W —> kW
- Divide the power in watts by 1000

$P = 60/1000 = .06\,kW$

$t = 12 \times 7 = 84\,hrs$

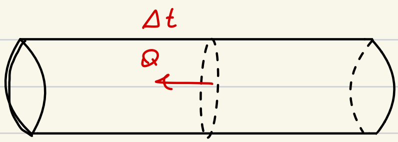
$E = P \times t = 5.04\,kwh$

$I = Q/t \qquad I = \Delta V/R \qquad R = \Delta V/I$

$A = 1C/1s \qquad \Delta V = IR \qquad 1ohm = 1volt/1A$

More Circuits (Resistors and Current):

Current (I):
- Defined as the rate of flow of charge past a cross section of a wire



$I = \frac{\Delta Q}{\Delta t}$

$\lim_{\Delta t \to 0} I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$

$I \propto \frac{1}{R}$

$I \propto \Delta V$

$\therefore I = \Delta V / R$

$\Delta x = V_d \Delta t$

$V = A \Delta x = A V_d \Delta t$

$\rightarrow V = \text{Volume}$

$Q = q_E \, n \, V$

n = Electron per unit volume

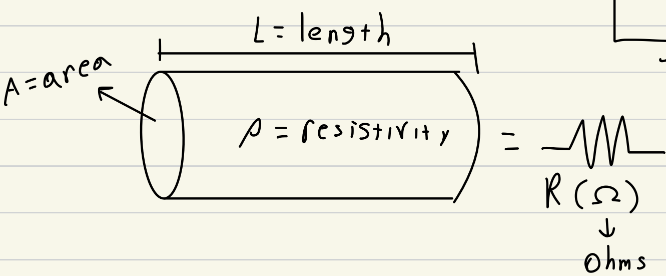
A = Cross section Area

$Q = q_E n A V_d \Delta t$

$I = Q/\Delta t = n q_E V_d A$

- The battery:
  - Provides/Gives potential energy to the charges as they move through

The Resistor:



$I \propto A$

$I = \frac{\Delta V}{R} \therefore R \propto \frac{1}{A}, \quad R \propto l$

$R = \rho \frac{l}{A}$

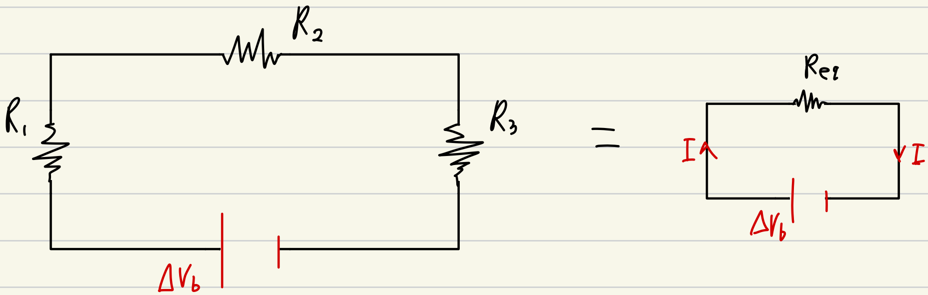
ρ = resistivity

Conductivity $\rightarrow \sigma = 1/\rho$

- Electrons dont just smoothly pass through a wire
- The electrons collide with the electrons in the copper atom (held on the outside), and have tons of collisions with them, 'zig-zagging' their way through the wire
- The **Drift Velocity** (mm/s) is the the average vector the electrons follow

Series Circuits:

Consider 3 resistors R1, R2, R3. We connect them in a series with a power supply (DELTA_V_b)



- Series = combination of resistors in a single loop circuit with the same current flowing through each element

$$I_1 = I_2 = I_3 = I$$

- In this current loop, the potential rise across the battery = sum of potential drop across the resistors

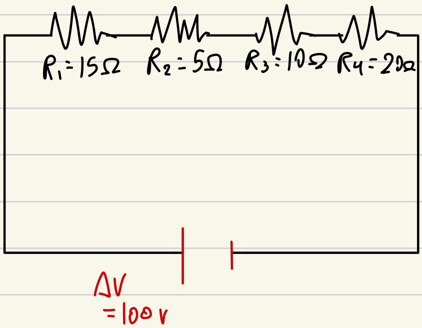
$$\Delta V_b = \Delta V_1 + \Delta V_2 + \Delta V_3$$

Ohm's: $\Delta V_1 = I_1 R_1$, $\Delta V_2 = I_2 R_2$, $\Delta V_3 = I_3 R_3$

$$\cancel{I} R_{eq} = I_1 R_1 + I_2 R_2 + I_3 R_3 \longrightarrow \cancel{I} R_1 + \cancel{I} R_2 + \cancel{I} R_3$$

$$R_{eq} = R_1 + R_2 + R_3$$

Four resistors 15, 5, 10 and 20 are connected in a Series with a 100V. Find the equivalent resistance, total current in the circuit and the current through and potential difference across each resistor in the combination.



$R_{eq} = R_1 + R_2 + R_3 + R_4$

$R_{eq} = 50\Omega$

$\Delta V_b : I_{tot} R_{eq}$

$100 = 50(I_{tot})$

$I_{tot} = 2A$

$I_{tot} = I_1 = I_2 \ldots = I_4 = 2A$

$\Delta V_1 = I_1 R_1 = 30V$

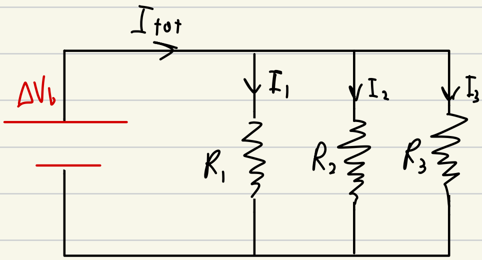
$\Delta V_2 = I_2 R_2 = 10V$

$\Delta V_3 = 20V$

$\Delta V_4 = 40V$

Parallel Circuits

- Circuit elements are connected such that they have the same potential different across them.



$\Delta V_1 = \Delta V_2 = \Delta V_3 = \Delta V_b$

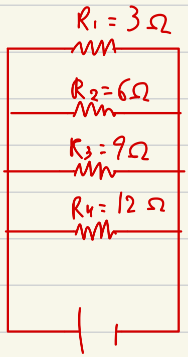
$I_{tot} = I_1 + I_2 + I_3$

$I_{tot} = \Delta V_b / R_{eq}$

$\cancel{\Delta V_b}/R_{eq} = \cancel{\Delta V_b}/R_1 + \cancel{\Delta V_b}/R_2 + \cancel{\Delta V_b}/R_3$

$1/R_{eq} = 1/R_1 + \ldots 1/R_3$

Ex. 4 resistors, 3,6,9,12 are connected in parallel across 36V battery. Find Eq Resistance and total current drawn. What is potential diff across and through each resistor?



$R_{eq} = (1/3 + 1/6 + 1/12 + 1/9)^{-1} = 1.44\ \Omega$

$\Delta V_b = I_{tot} R_{eq}$

$I_{tot} = 25A$

$\Delta V_1 = \ldots \Delta V_4 = 36V$

$\Delta V_1 = I_1 R_1 \rightarrow I_1 = 12A$

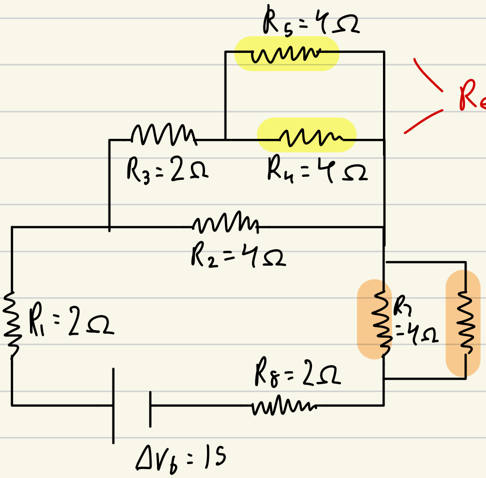
$I_2 = 6A$

$I_3 = 4A$

$I_4 = 3A$

Complex Series and Circuits

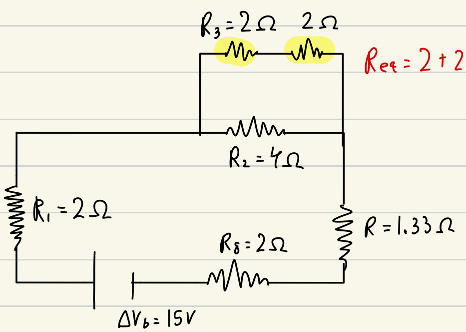
Find equivalent resistance and total current, and current through and potential difference across each resistor in the circuit below.



$R_{eq} = (1/4 + 1/4)^{-1} = 2\,\Omega$

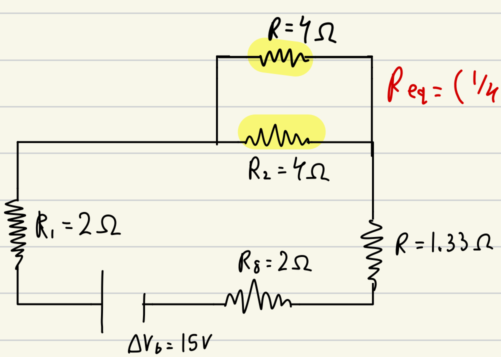
$R_{eq} = (1/2 + 1/4)^{-1} = 1.33\,\Omega$

=

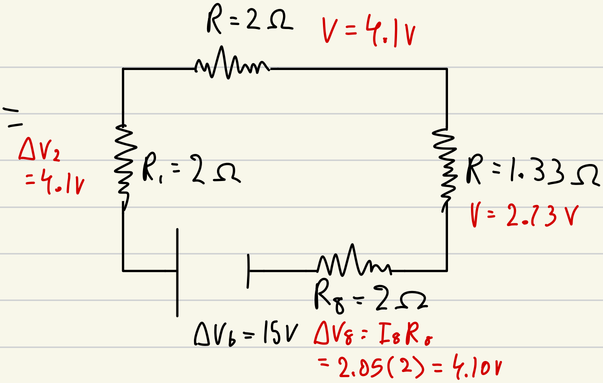


$R_{eq} = 2 + 2 = 4\Omega$

=



$R_{eq} = (1/4 + 1/4)^{-1} = 2\Omega$



$R_{eq} = 7.33\Omega$

$\Delta V_b = 15V$

$\Delta V_b = I_{tot} R_{eq}$

$15 = I_{tot}(7.33)$

$I_{tot} = 2.05A$

$I = \Delta V / R$

*Go back & find vals

$\Delta V_1 = 4.1V$	$I_1 = 2.05A$
$\Delta V_2 = 4.1V$	$I_2 = 1.025A$
$\Delta V_3 = 2.05V$	$I_3 = 1.025A$
$\Delta V_4 = 2.05V$	$I_4 = .5125A$
$\Delta V_5 = 2.05V$	$I_5 = .5125A$
$\Delta V_6 = 2.73V$	$I_6 = 1.365A$
$\Delta V_7 = 2.73V$	$I_7 = .6825A$
$\Delta V_8 = 4.1V$	$I_8 = 2.05A$

Ex problem

A 30cm long copper wire is connected to a 250V power supply. An ammeter measures a current of 20mA passing through the wire. The resistivity of copper is 1,68 x 10^-8.
a) Resistance of wire
B) what is diameter of wire
c) how much charge flows through wire in hour
D) how many electrons pass
E) how much energy dissipated in kwh

$\Delta V = 250V$
$I = 20mA$
$\rho = 1.68 \times 10^{-8} \Omega m$
$l = .2 m$

a) $\Delta V = IR$
$R = 250/20 \times 10^{-3} = \boxed{12500 \Omega}$

b) $R = \rho L/A$
$12500 = 1.68 \times 10^{-8}(.2)/A$
$A = 2.69 \times 10^{-13} m^2$
$A = \pi r^2 \rightarrow r = 2.93 \times 10^{-7} m$
$\boxed{d = 5.86 \times 10^{-7} m}$

c) $I = Q/t$
$20 \times 10^{-3} = q/3600$
$q = 72C$

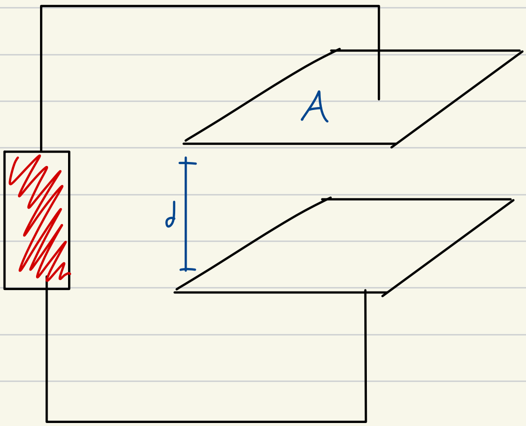
d) $72/1.6 \times 10^{-19} = 4.5 \times 10^{20} e^-$

e) $P = \Delta V I = 250 \times 20 \times 10^{-3} = 5W$
$P = E/t$
24hrs
$5 = E/86400$
$E = 432000J$
kwh: $P = 5w = .005kw$
$t = 24hr$
$E = Pt = .005 \times 24 = .12 kwh$

Capacitors and Circuits

Capacitance (C):

- Amount of charge it can hold



$C = Q/\Delta V$

*Always keeps the same proportion
Measured in Farads (F)

$\vec{E} = \frac{\sigma}{\epsilon_0}$

$\sigma \rightarrow$ Charge/unit area on a plate of capacitor

$\sigma = Q/A$

$\epsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

$$\boxed{\begin{array}{c} C = Q/\Delta V \\ C = \frac{\epsilon_0 A}{d} \\ \vec{E} = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \\ \Delta V_c = \frac{Q d}{A \epsilon_0} \\ \epsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2 \end{array}}$$

$\vec{E} = \frac{\Delta V_c}{\Delta x} = \frac{\Delta V_c}{d}$

$\frac{\sigma}{\epsilon_0} = \frac{\Delta V_c}{d}$

$\Delta V_c = \frac{\sigma (d)}{\epsilon_0}$

$\Delta V_c = \frac{Q\, d/A}{\epsilon_0}$

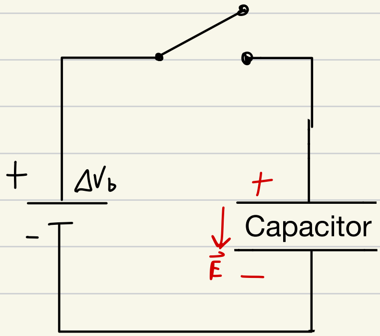
$\Delta V_c = \frac{Q\, d}{A \epsilon_0}$

$C = Q/ Qd/_{A\epsilon_0} = \frac{\epsilon_0 A}{d}$

Alt Formulas:

$U_c = 1/2\, Q\, V$

$U_c = 1/2\, C \Delta V^2$



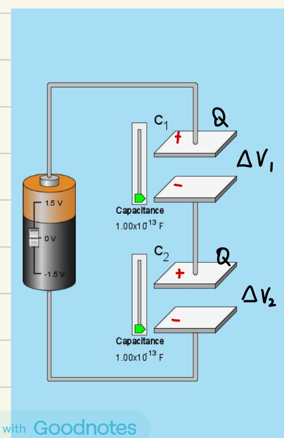
Asymptotic behavior:
- when the capacitor is initially uncharged, and just begins to charge (immediately after the switch is closed), **the capacitor acts like a bare wire (no resistance)(charge flows freely)**
- After the switch has been closed for a long time, **the capacitor is fully charged and acts like an open switch (no current flows in the circuit).**

Dielectric:
- An object you put in between the capacitor (ex. Insulator)
- The capacitor polarizes particles in the object causing an opposite E-field
- In other words a dielectric will weaken field of capacitor
- Has a dielectric strength (max E-field before it breaks down)

$\epsilon \rightarrow$ Permitivity of dielectric

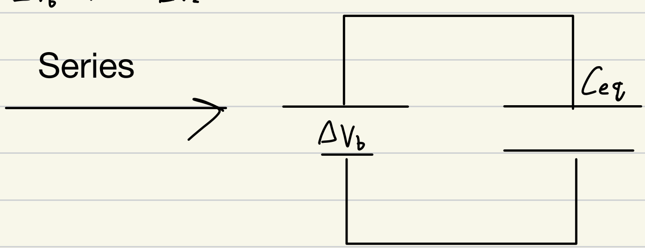
Dielectric const. $k = \frac{\epsilon}{\epsilon_0}$

$C = \epsilon \frac{A}{D} \quad || \quad C = k\epsilon_0 \frac{A}{D}$



$\Delta V_b = \Delta V_1 + \Delta V_2$

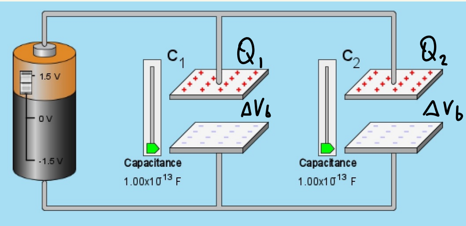
Series



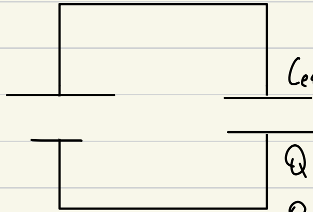
$C_{eq} = \frac{Q}{\Delta V_B} \rightarrow \Delta V_b = \frac{Q}{C_{eq}}$

$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} \rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$

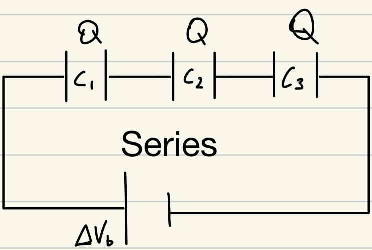


Parallel

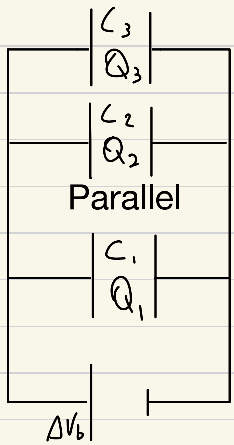


$$Q_{tot} = Q_1 + Q_2$$
$$Q_{tot} = C_{eq}\Delta V_b$$
$$Q_{tot} = C_1\Delta V_1 + C_2\Delta V_2$$
$$C_{eq}\Delta V_b = \Delta V_b (C_1 + C_2)$$
$$C_{eq} = C_1 + C_2$$
$$C_{eq} = \sum_{i=1}^{n} C_i$$

Quick Overview:

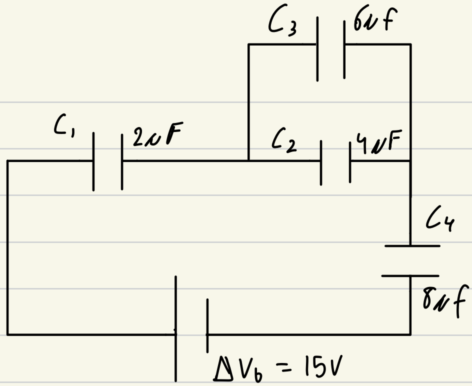


$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$
$$\Delta V_b = \Delta V_1 + \Delta V_2 + \Delta V_3$$
$$C = Q/\Delta V$$



$$C_{eq} = C_1 + C_2 + C_3$$
$$\Delta V_1 = \Delta V_2 = \Delta V_3$$
$$C = Q/\Delta V$$

Ex.



a) Find equivalent capacitance of the combination
b) Find individual potential difference and charge for each capacitor

a) $C_{23} = C_2 + C_3 = 10\mu f$

$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4}\right)^{-1} = 1.38\mu f$

b) $Q_{tot} = C_{eq}\Delta V = 20.7\mu C$

$Q_1 = Q_{23} = Q_4 = 20.7\mu C$

$\Delta V_1 = Q_1/C_1 = \frac{20.7}{2} = 10.35V$

$\Delta V_4 = Q_4/C_1 = \frac{20.7}{8} = 2.59V$

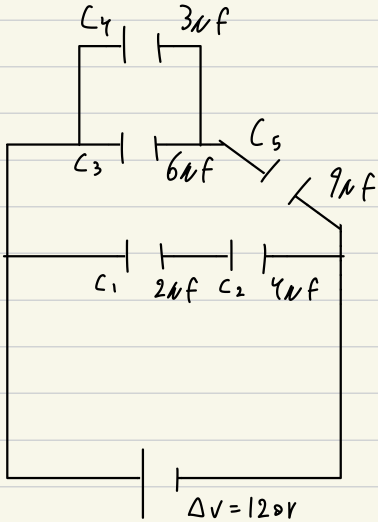
$\Delta V_{23} = Q_{23}/C_{23} = \frac{20.7}{10} = 2.07V$

$\Delta V_2 = \Delta V_3 = \Delta V_{23} = 2.07V$

$Q_2 = \Delta V_2 C_2 = 2.07(4) = 8.28\mu C$

$Q_3 = \Delta V_3 C_3 = 2.07(6) = 12.42\mu C$

Ex.



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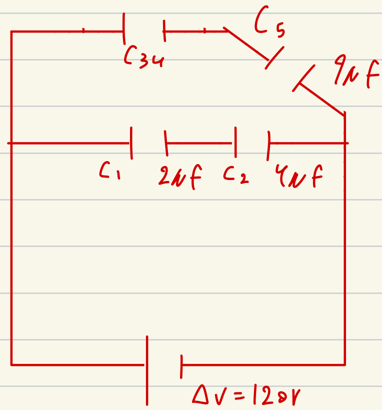
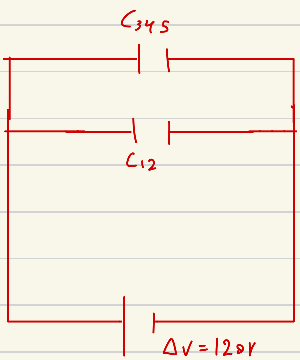


$C_{34} = C_3 + C_4 = 6 + 3 = 9nf$

$\equiv$



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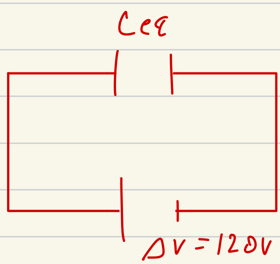


$C_{345} = \left(\frac{1}{C_{34}} + \frac{1}{C_5}\right)^{-1} = 4.5nf$

$C_{12} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = 1.33nf$

$C_{eq} = C_{345} + C_{12} = \underline{5.83nf}$

$Q_{tot} = C_{eq}(\Delta V) = \underline{699.6nC}$

$\Delta V_{12} = \Delta V_{345} = \underline{120V}$

$Q_{12} = C_{12}\Delta V_{12} = 159.6nC$

$Q_{345} = C_{345}\Delta V_{345} = 540nC$

$Q_{12} = Q_1 = Q_2 = 159.6nC$

$Q_{34} = Q_5 = 540nC$

$\Delta V_{34} = \frac{Q_{34}}{C_{34}} = 60V$

$\Delta V_3 = \Delta V_4 = 60V$

$Q_3 = C_3\Delta V_3 = 360nC$

$Q_4 = C_4\Delta V_4 = 180nC$

$\Delta V_1 = \frac{Q_1}{C_1} = 79.8V$

$\Delta V_2 = \frac{Q_2}{C_2} = 39.9V$

$\Delta V_5 = \frac{Q_5}{C_5} = 60V$

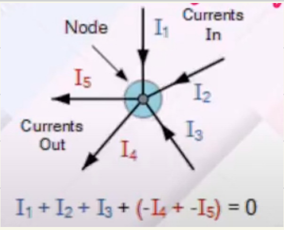
Kirchhoff's Rule:

Frequently, complex circuits cannot be reduced simply by using our understanding because of various reasons (ex. Not parallel or series)

- Kirchhoff's Rule is a tool to simplify circuits

Junction Rule:

- The sum of all currents entering a junction must equal sum of all currents leaving the junction

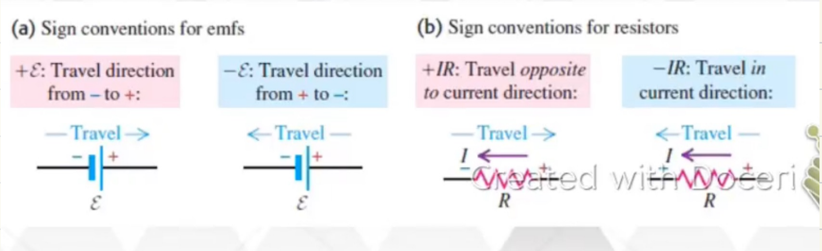


$$\sum I_{in} = \sum I_{out}$$

- Application of law of conservation of charge
- Current is flow of charge

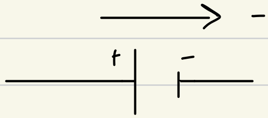
Loop Rule:

- The sum of the changes in potential at different points around any closed circuit path or loop must be zero
- Application of conservation of energy
- In a closed loop, PE supplied by EMF must be transformed into other types of energy
- Sign Conventions:

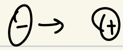


$$\sum \Delta V = 0$$

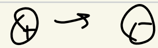
$-(qE\Delta x)$



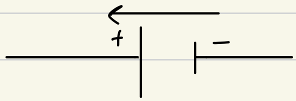
$-\Delta V$



$\Delta U_E = +$

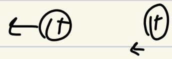


$\Delta U_E = -$

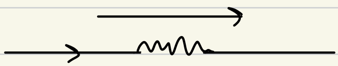


$+\Delta V$

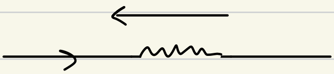
$\Delta U_E = -$



$\Delta U_E = -$



$-IR$

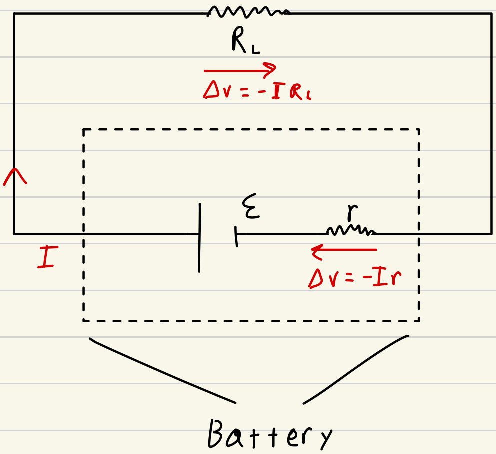


$+IR$

Batteries, Voltmeters and Ammeters

Internal resistance = Resistance of better

- When electrons go through the battery they lose and gain energy



$\varepsilon \rightarrow emf \rightarrow$ Electromotive Force

$R_{eq} = R_L + r$

$I = \varepsilon / R_{eq} = \varepsilon / (R_L + r)$

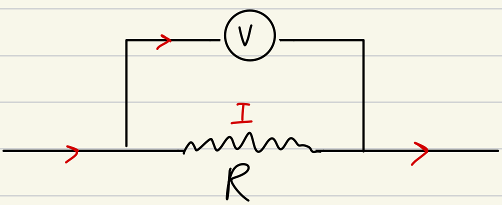
Loop Rule:

$\varepsilon - IR_L - Ir = 0$

$\varepsilon - Ir = IR_L \longrightarrow$ Terminal Voltage

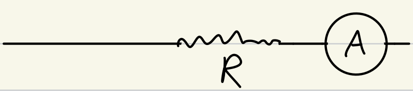
$V_T = \varepsilon - Ir$

- Ammeter connects in series because current needs to be same
- Voltmeter connects in parallel because voltage needs to be same

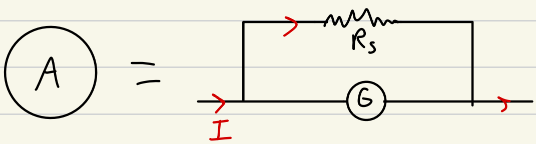


*Voltmeter must have a huge resistance to negate the fact that current splits (therefor changing the voltage)

- We want to limit the current that goes into the V-meter



*Similarly, ammeter must have a super low resistance to negate the current drop (due to resistance)



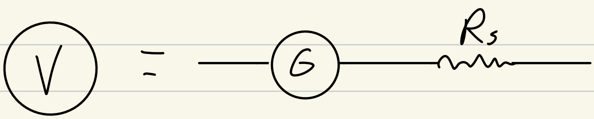
$\Delta V_s = \Delta V_G$

$I_s R_s = I_G R_G$

$(I_{max} - I_s) R_s = I_G R_G$

$R_s = I_G R_G / (I_{max} - I_G)$

Fs = R-Shunt
G = Galvanameter (can only measure very small amounts of current)
Resistor put in place to not overload the galvanameter



- Rs is bigger

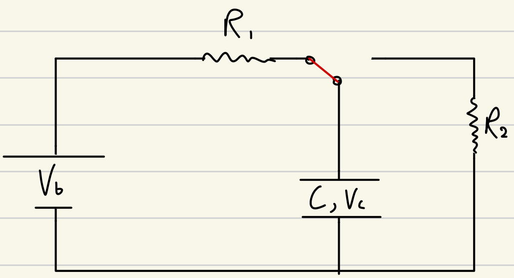
$\Delta V_{max} = \Delta V_G + \Delta V_s$

$= I_G R_G + I_G R_s$

$= I_G (R_G + R_s)$

$R_s = \Delta V_{max} - I_G R_G / I_G$

Capacitors and Resistors



Loop Rule)

$$V_b - V_c - IR_1 = 0$$

$$V_b - Q/C - \frac{dQ}{dt}R_1 = 0$$

$$\frac{dQ}{dt} = \frac{V_b}{R_1} - \frac{Q}{R_1C}$$

$$\left(\frac{dQ}{dt} = \frac{V_b}{R_1} - \frac{Q}{R_1C}\right)dt$$

$$\frac{dQ}{dt}\cdot dt = \left(\frac{V_b}{R_1} - \frac{Q}{R_1C}\right)dt \rightarrow \text{Time int.}$$

$$dQ = \left(\frac{V_b}{R_1} - \frac{Q}{R_1C}\right)dt$$

$$\frac{dQ}{\left(\frac{V_b}{R_1} - \frac{Q}{R_1C}\right)} = dt$$

At t=0s, Q=0C (b/c the capacitor was uncharged before circuit activated)

$$\int \frac{dQ}{\left(\frac{V_b}{R_1} - \frac{Q}{R_1C}\right)} = \int dt$$

$$u = \frac{V_b}{R_1} - \frac{Q}{R_1C}$$

$$du = -dQ/R_1C$$

$$-R_1C\,du = dQ$$

$$\int \frac{-R_1C\,du}{u} \rightarrow -R_1C\int\frac{du}{u} = t + \eta$$

$$-R_1C\ln(u) = t + \eta$$

$$\ln\left(\frac{V_b}{R_1} - \frac{Q}{R_1C}\right) = \frac{-t}{R_1C} - \frac{\eta}{R_1C}$$

At t=0

$$\ln\left(\frac{V_b}{R_1}\right) = \frac{-\eta}{R_1C} \rightarrow \eta = -R_1C\ln\left(\frac{V_b}{R_1}\right)$$

$$\ln\left(\frac{V_b}{R_1} - \frac{Q}{R_1C}\right) = \frac{-t}{R_1C} + \frac{R_1C\ln\left(\frac{V_b}{R_1}\right)}{R_1C}$$

$$\ln\left(\frac{V_b}{R_1} - \frac{Q}{R_1C}\right) = \frac{-t}{R_1C} + \ln\left(\frac{V_b}{R_1}\right)$$

$$\ln\left(\frac{V_b}{R_1} - \frac{Q}{R_1C}\right) - \ln\left(\frac{V_b}{R}\right) = \frac{-t}{R_1C}$$

$$\ln\left(\frac{\frac{V_b}{R_1} - \frac{Q}{R_1C}}{V_b/R_1}\right) = -\frac{t}{R_1C}$$ (Exponentiate to e to get rid of ln)

$$\frac{\frac{V_b}{R_1} - \frac{Q}{R_1C}}{V_b/R_1} = e^{\frac{-t}{R_1C}}$$

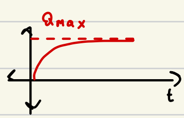
$$\frac{V_b}{R_1} - \frac{Q}{R_1C} = \frac{V_b}{R_1}e^{-t/R_1C}$$

$$+\frac{Q}{R_1C} = \frac{V_b}{R_1} - \frac{V_b}{R_1}e^{-t/R_1C}$$

$$Q = R_1C\cdot\frac{V_b}{R_1} - R_1C\cdot\frac{V_b}{R_1}e^{-t/R_1C}$$

$$\boxed{Q(t) = \underbrace{CV_b}_{\text{Q-max=CVb}}\left(1 - e^{-t/R_1C}\right)} \rightarrow \boxed{R_1C = \tau} \rightarrow$$

Time constant (bigger the time constant, the flatter the curve)



$t = \tau$ Time constant tells us how quick capacitor would charge

Charge:

$$Q(t) = Q_{max}\left(1 - e^{-\frac{RC}{R_1C}}\right)$$

$$= Q_{max}\left(1 - e^{-1}\right)$$

$$Q(t) = .632\, Q_{max} \longleftrightarrow V(t) = .632\, V_b$$

Voltage:

$$\Delta V(t) = \Delta V_b\left(1 - e^{-t/R_1C}\right)$$

$$V(t) = V_b\left(1 - e^{-t/R_1C}\right)$$

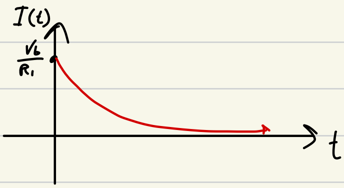
Current:

$$I(t) = \frac{dQ}{dt} = \frac{d}{dt}\left\{Q_{max}\left(1 - e^{-t/R_1C}\right)\right\}$$

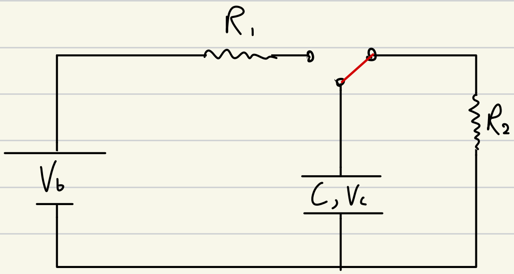
$$I(t) = \frac{Q_{max}}{R_1C}\, e^{-t/R_1C}$$

$$= \frac{\Delta V_b}{R_1 C}\, e^{-t/R_1C}$$

$$I(t) = \frac{V_b}{R_1}\, e^{-t/R_1C} \longrightarrow$$



Flipped switch:



$$R_2 = R$$
$$V_c = -V_R$$
$$\frac{Q}{C} = -R\frac{dQ}{dt}$$
$$\frac{dQ}{dt} = -Q/RC$$
$$\frac{dQ}{Q} = -\frac{dt}{RC}$$
$$\int \frac{dQ}{Q} = \int \frac{-dt}{RC}$$

At t=0, Q=Qmax

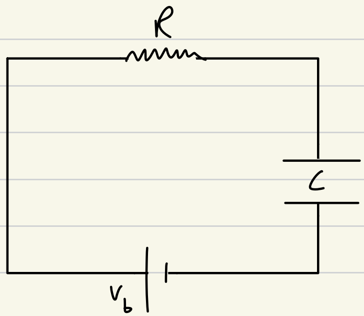
$$\int \frac{dQ}{Q} = \int \frac{-dt}{RC}$$
$$\ln(Q) = \frac{-t}{RC} + \eta$$

At t=0:

$$\ln(Q_{max}) = \eta$$

$$\ln Q = \frac{-t}{RC} + \ln Q_{max}$$
$$\ln Q - \ln Q_{max} = \frac{-t}{RC}$$
$$\ln\left(Q/Q_{max}\right) = e^{-t/RC}$$
$$Q(t) = Q_{max}\, e^{-t/RC}$$

Summary:



$Q(t) = Q_{max}(1-e^{-t/RC}) \rightarrow$ Charging

$V_c(t) = V_b(1-e^{-t/RC})$

$Q(t) = Q_{max}e^{-t/RC} \longrightarrow$ Discharging

RC = Time Constant $= \tau$

At t=0, when circuit is just turned on, Q on capacitor is 0C

$t=0, \quad V_R = V_b \quad \& \quad V_c = 0$

$t \to \infty, \quad V_c = V_b \quad \& \quad V_R = 0$

Charging:

$Q(\tau) = Q_{max}(1-e^{-1})$

$= .632\, Q_{max}$

Discharge:

$Q(t) = Q_{max}e^{-t/RC}$

$Q(\tau) = Q_{max}/e = .368\, Q_{max}$

Energy of Capacitor: $E = 1/2QV$ or $E=1/2CV^2$